# Bio-electric HW #1: (Sept. 24th, 2014)

## Problem #1

Here a small cylindrical electrode  $(r=1, h = 2)$  sits inside a bigger sphere  $(r=20)$ . The sphere is grounded, the space in between has a conductivity of  $0.25\frac{S}{m}$ , and the cylindrical electrode emits a current density of  $1\frac{A}{m^2}$  from its sides. The model is shown in figure 1. Since the sphere is grounded and the cylinder acts as a current source, it was expected that the resultant potential throughout the medium would decrease radially outward from the source. This was confirmed by the computed study. Figure 3 shows the XY plane slice which best represents the decreasing potential as a function of radius. The potential is highest directly outside the surface of the electrode and then smallest  $(0)$  at the surface of the sphere.



Figure 1: 3D Model View



Figure 2: 3D Potential View



Figure 3: XY plane Potential

Here the large outer cylinder is not grounded. Inside it are two spherical electrodes, the lower of which acts as a current source with a current density of 1... while the lower acts as a current sink with a current density of -1. It was expected that the magnitude of the potential would be largest nearest the surface of either electrode. Although the electrodes would be opposite in sign, their identical size and current density magnitudes would mean the gradient of potential was expected to be symmetrical in magnitude. This was confirmed by the computed study which showed an equal positive and negative range, most negative near the negative electrode and symmetrically positive near the positive electrode. Any point equidistant from the two electrodes (ie the center plane of the cylinder) was expected to be 0 as it would cancel the equal contributions of each electrode. This was confirmed by the study via the XY plane slice which showed all green (0 potential).



Figure 4: 3D Model View



Figure 5: 3D Potential View



Figure 6: XY plane Potential



Figure 7: YZ plane Potential



Figure 8: XZ plane Potential

Now the setup is identical to problem #2, except that the outer cylinder has been grounded. This was not expected to have any effect on the symmetrical nature of the potential gradient, but have a dramatic effect on the drop off of potential radially out from either electrode. This was expected because of the study from the model in problem  $#1$ where a grounded outer shell caused a drop off in potential radially outward from the electrode. This expectation was confirmed via the study for the model in problem  $#3$ . The potential gradients are still symmetrical, and 0 anywhere that is equidistant from both electrodes. However the potential drops off much quicker as a result of the grounded shell. It's as if one simply combined the isolated effects of model  $\#1$  and model  $\#2$ .



Figure 9: 3D Potential View



Figure 10: XY plane Potential



Figure 11: YZ plane Potential



Figure 12: XZ plane Potential

Now the model returns to having an ungrounded shell, but the upper electrode which previously served as a sink (negative current density) is now grounded instead. This was not expected to change the behavior of the potential gradient in function, but was expected to have effects on the value of potential at every point. If the potential from model #2 at any point in the cylinder was the combination of the effects of both electrodes... being evenly balanced when equidistant from both electrodes, positive when closer to the positive electrode and negative when closer to the negative electrode... then replacing the negative electrode with a ground shouldn't change the gradient of the potential. It should simply perform a linear shift on the potential value at every point by the amount the magnitude of the negative portion previously contributed by the negative electrode that is now removed. Ie: if the value of potential at some point was  $x+y$  in problem  $#2$ , where x was the contribution from the positive electrode and y was the contribution from the negative electrode, then the potential in model  $#4$  would be  $x+0$  because the negative electrode is now a ground. This is confirmed by the figures for model#4 which show the exact same distribution as model  $#2$ , but the range of values is from  $[0:2*max]$  instead of  $[-max:max]$ .



Figure 13: 3D Potential View



Figure 14: XY plane Potential



Figure 15: YZ plane Potential



Figure 16: XZ plane Potential

Problems 2-4 demonstrated that the presence of a ground in a system with current density can dramatically affect the potential gradient. Additionally currents will counterbalance each other to the best ability of their magnitudes. The two effects on potential can be present simultaneously as if each is independent of the other.