Problem 1:

Contents of the Differential Eqn Function for ODE45:

```
function Dydt = hh_diffeqn(t,y)

%%%%%%%%%%%%%%%%%%%  Define Model Parameters   %%%%%%%%%%%%%%%%%%%%%%%%%%%
I_s = 0;             % injected stimulation current
V_rest = -65  ;      % resting membrane voltage in mV
T = 6.3;             % temperature in degrees C
T_k = 273.15+T;      % temperature in Kelvin


V = y(1,1);          % membrane voltage
m = y(2,1);          % m (Gate variable)
n = y(3,1);          % n (Gate variable)
h = y(4,1);          % h (Gate variable)

v_m = V-V_rest;      % difference between V and V_rest

gNa = 120;           % Na+ max conductance in mS/cm^2
gK = 36;             % K+  max conductance in mS/cm^2
gL = 0.3;            % Leak conducation    in mS/cm^2

conc_Na_ext = 490;  % Extracellular Na+ concentration (mmol/L)
conc_Na_int = 50;   % Intracellular Na+ concentration (mmol/L)
conc_K_ext  = 20;   % Extracellular K+  concentration (mmol/L)
conc_K_int  = 400;  % Intracellular K+  concentration (mmol/L)

R = 8.314;           % Ideal Gas Cosntant
F = 9.6485*10^4;     % Faraday's Constant

E_Na = 1000*(R*T_k/F)*log(conc_Na_ext/conc_Na_int);  % Na+ Nernst Potential (mV)
E_K  = 1000*(R*T_k/F)*log(conc_K_ext/conc_K_int);    % K+  Nernst Potential (mV)
E_L  = -50;                                          % Leak Nernst Potential(mV)

C = 1.0;             % membrane capacitance in uF/cm^2

k = 3^(0.1*T-0.63); % temperature dependence factor to affect kinetics


%%%%%%%%%%%%%%%%%%%%   Calculate Rate Constants    %%%%%%%%%%%%%%%%%%%%%%%%

alpha_m = .1*(25-v_m)./(exp((25-v_m)/10)-1);    % Rate Constant
alpha_h = .07*exp(-v_m/20);                     % Rate Constant
alpha_n = .01*(10-v_m)./(exp((10-v_m)/10)-1);   % Rate Constant
beta_m  = 4*exp(-v_m/18);                       % Rate Constant
beta_h  = 1/(exp((30-v_m)/10)+1);               % Rate Constant
beta_n  = .125*exp(-v_m/80);                    % Rate Constant

%%%%%%%%%%%%%%%%%%%%   Calculate Derivatives    %%%%%%%%%%%%%%%%%%%%%%%%%%%

dVdt  = (1/C).*(I_s-gNa*m^3*h*(V-E_Na)-gK*n^4*(V-E_K)-gL*(V-E_L));
dmdt = (-(alpha_m+beta_m)*m+alpha_m)*k;
dndt = (-(alpha_n+beta_n)*n+alpha_n)*k;
dhdt = (-(alpha_h+beta_h)*h+alpha_h)*k;


%%%%%%%%%%%%%%%%%%%%   Initialize & Populate Output   %%%%%%%%%%%%%%%%%%%%%
Dydt = zeros(4,1);
Dydt(:,1) = [dVdt; dmdt; dndt; dhdt];
end
```

Contents of the Run Script:

```
close all; clear all; clc;
%%%%%%%%%%%%%%%%%%%  Define Relevant Model Parameters   %%%%%%%%%%%%%%%%%%%
T = 6.3;            % temperature in degrees C
gNa = 120;          % Na+ max conductance in mS/cm^2
gK = 36;            % K+  max conductance in mS/cm^2
conc_Na_ext = 490;  % Extracellular Na+ concentration (mmol/L)
conc_Na_int = 50;   % Intracellular Na+ concentration (mmol/L)
conc_K_ext  = 20;   % Extracellular K+  concentration (mmol/L)
conc_K_int  = 400;  % Intracellular K+  concentration (mmol/L)
R = 8.314;          % Ideal Gas Cosntant
F = 9.6485*10^4;    % Faraday's Constant
E_Na = 1000*(R*T/F)*log(conc_Na_ext/conc_Na_int);    % Na+ Nernst Potential (mV)
E_K  = 1000*(R*T/F)*log(conc_K_ext/conc_K_int);      % K+  Nernst Potential (mV)

%%%%%%%%%%%%%%%%%%%     Define Initial Conditions        %%%%%%%%%%%%%%%%%%
V_rest  = -65;                  %resting membrane voltage
m_0     = 0.05;                 %initial rate constant m value
n_0     = 0.32;                 %initial rate constant n value
h_0     = 0.6;                  %initial rate constant h value
y_0 = [V_rest; m_0; n_0; h_0];  %initial conditions in one array

%%%%%%%%%%%%%%%%%%%     Define ODE Parameters        %%%%%%%%%%%%%%%%%%%%%%
dt = [0,20];                    %time span for the ode45 function to use
options = odeset('RelTol',1e-4,'AbsTol',[1e-8,1e-8,1e-8,1e-8],'MaxStep',0.01);

%%%%%%%%%%%%%%%%%%%     Run the ODE45 Simulation     %%%%%%%%%%%%%%%%%%%%%%
[t,y] = ode45('hh_diffeqn',dt,y_0,options);

V = y(:,1);             %pull out resultant voltage
M = y(:,2);             %pull out resultant m
N = y(:,3);             %pull out resultant n
H = y(:,4);             %pull out resultant h

%%%%%%%%%%%%%%%%%%%     plot v,m, n and h     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure;
subplot(3,2,1); plot(t,V); title('Voltage Time Series'); xlabel('Time (ms)'); ylabel('Voltage (mV)');
subplot(3,2,2); plot(t,M); title('M Gate'); xlabel('Time (ms)'); ylabel('Probability in Open Conformation');
subplot(3,2,3); plot(t,H); title('H Gate'); xlabel('Time (ms)'); ylabel('Probability in Open Conformation');
subplot(3,2,4); plot(t,N); title('N Gate'); xlabel('Time (ms)'); ylabel('Probability in Open Conformation');

%%%%%%%%%%%%%%%%%%%%   Calculate Currents      %%%%%%%%%%%%%%%%%%%%%%%%%%%%
I_K = gK.*N.^4.*(V-E_K);
I_Na = gK.*M.^3.*H.*(V-E_Na);

%%%%%%%%%%%%%%%%%%%        plot currents      %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%figure;
subplot(3,2,5); plot(t,I_K); title('Potassium Current (K+)'); xlabel('Time (ms)'); ylabel('Current (mA)');
subplot(3,2,6); plot(t,I_Na); title('Sodium Current (Na+)'); xlabel('Time (ms)'); ylabel('Current (mA)');
```

Problem 2: In order to ensure the membrane had properly settled, a time scale of [0:60]ms was used, with the resultant steady state being pulled off the 60ms time mark. Steady state membrane potential with 0 injected stimulus current was found to be -60.34mV after 9-10 runs. The assumed resting membrane potential before each integrated solution followed the pattern:
$V_{rest} = -65, -62.03, -61, -60.61, -60.45, -60.39, -60.36, -60.35, -60.34, -60.34$

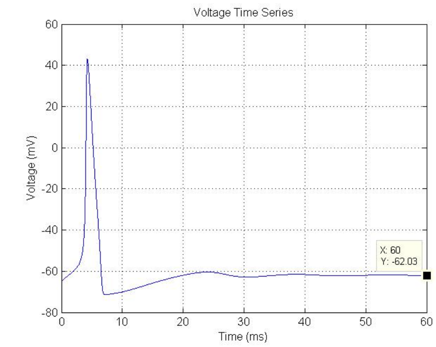

Figure 1: Membrane Voltage under assumption that $V_{rest} = -65mV$.

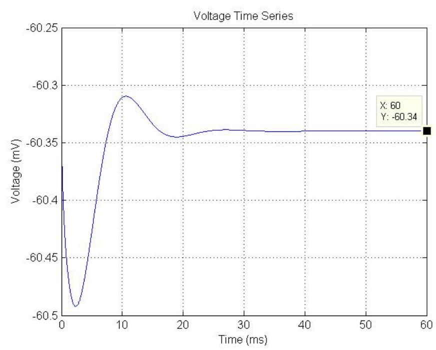

Figure 2: Membrane Voltage under finalized assumption that $V_{rest} = -60.34mV$.

Problem 3:
The minimum amplitude of a square stimulation current pulse of width 0.35ms (onset at $t = 1ms$ and removed at $t = 1.35ms$) was found to be between 19.4 and 19.5, which at the accuracy of 1 decimal place means a minimum stimulation current of 19.5.

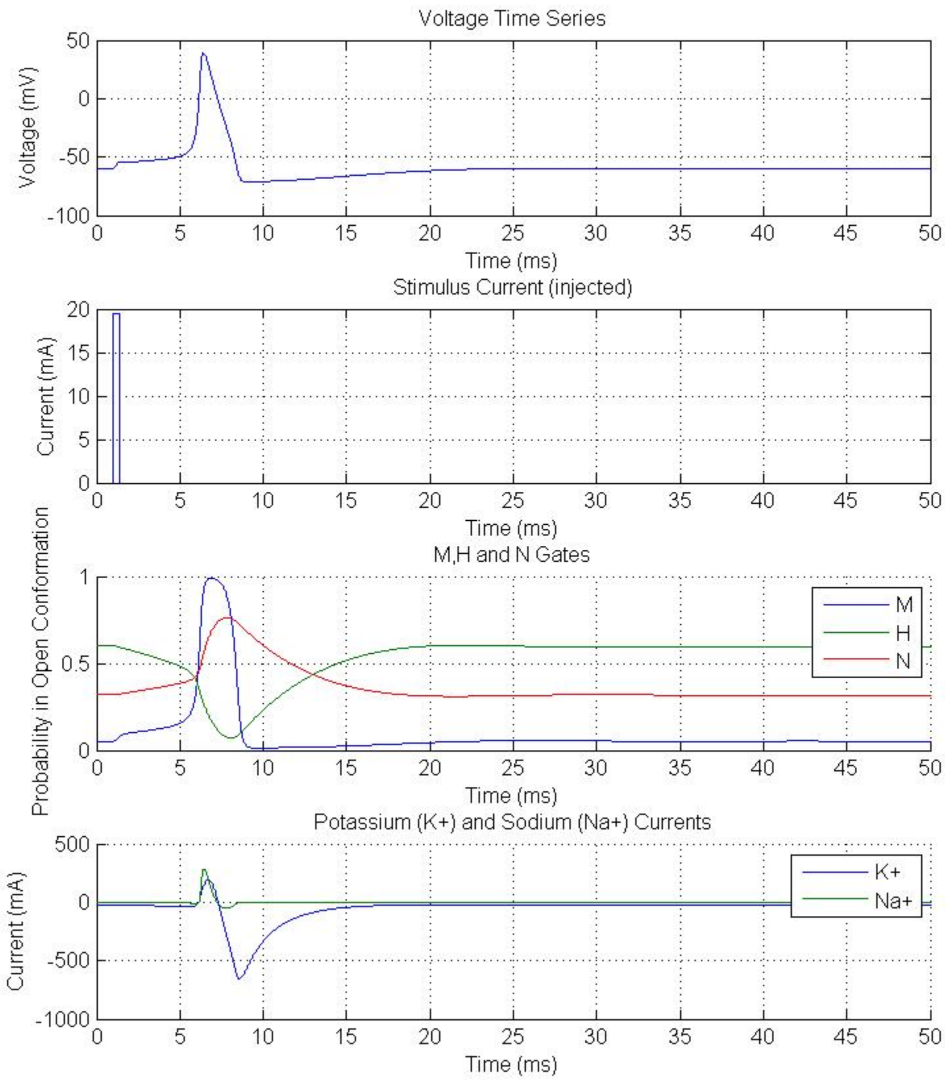

Figure 3: Minimum Stimulation current to elicit an Action Potential.

Problem 4-A

For equally sized pulses, the minimum delay between the removal of the first pulse and the onset of the second pulse was found to be between 19.25 ms. Put in other words, the time between the onsets was $35 + 19.25 = 54.25ms$.

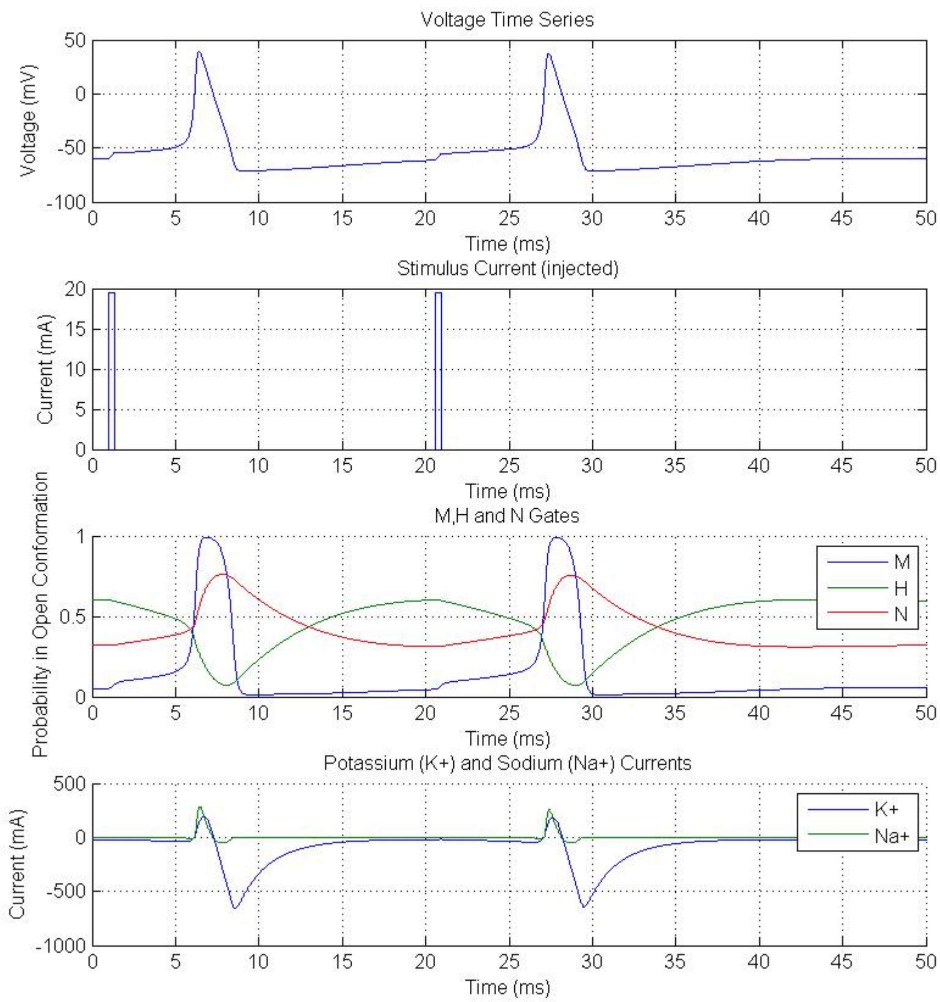

Figure 4: Minimum delay between two consecutive APs induced by the same current pulse.

Problem 4-B:
Bumping the magnitude of the second pulse up by a factor of 3 meant that the minimum delay between the removal of the first smaller pulse and the onset of the second larger pulse was found to be be 13.8 ms.

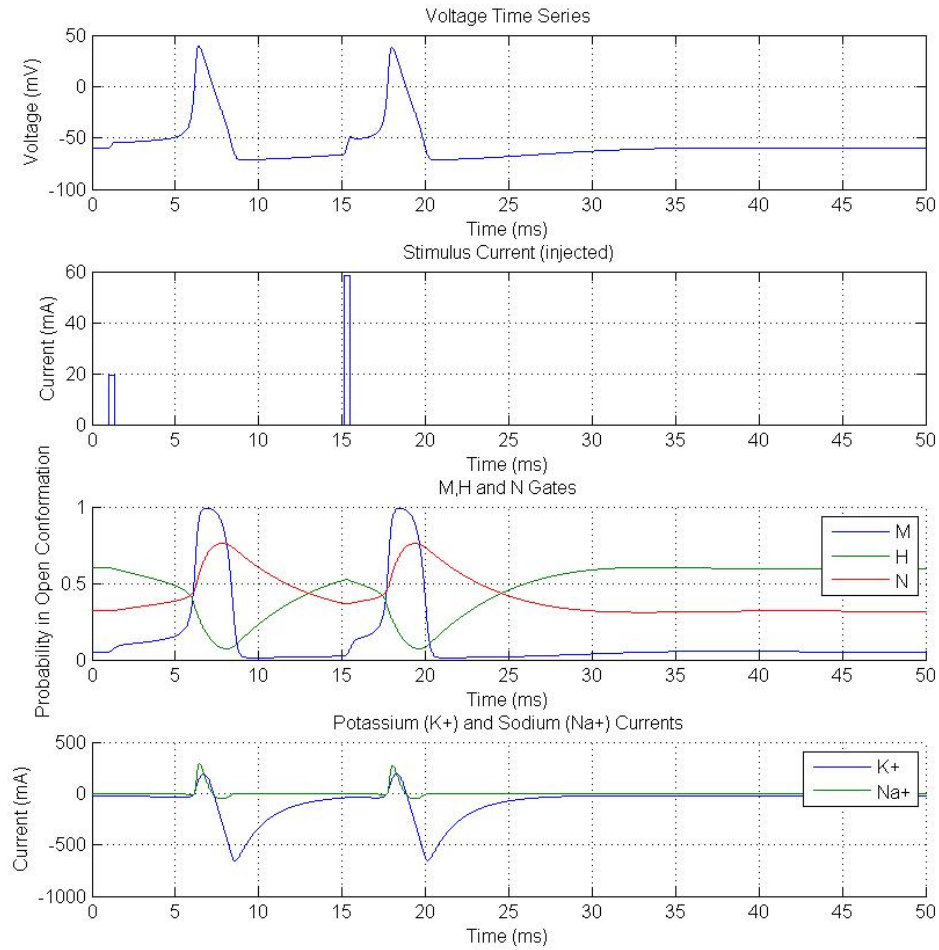

Figure 5: Minimum delay between two consecutive APs induced by the different current pulses.

Problem 5:
When we stimulated with a single pulse of width .35ms, the minimum current required to elicit an AP was 19.5. If we stimulate with a single negative current pulse with amplitude equal to 3×19.5, coincidentally the minimum width of the negated stimulus required to elicit an AP was between .34 and .35. At the accuracy of 2 decimal places, .35ms served as a minimum width.

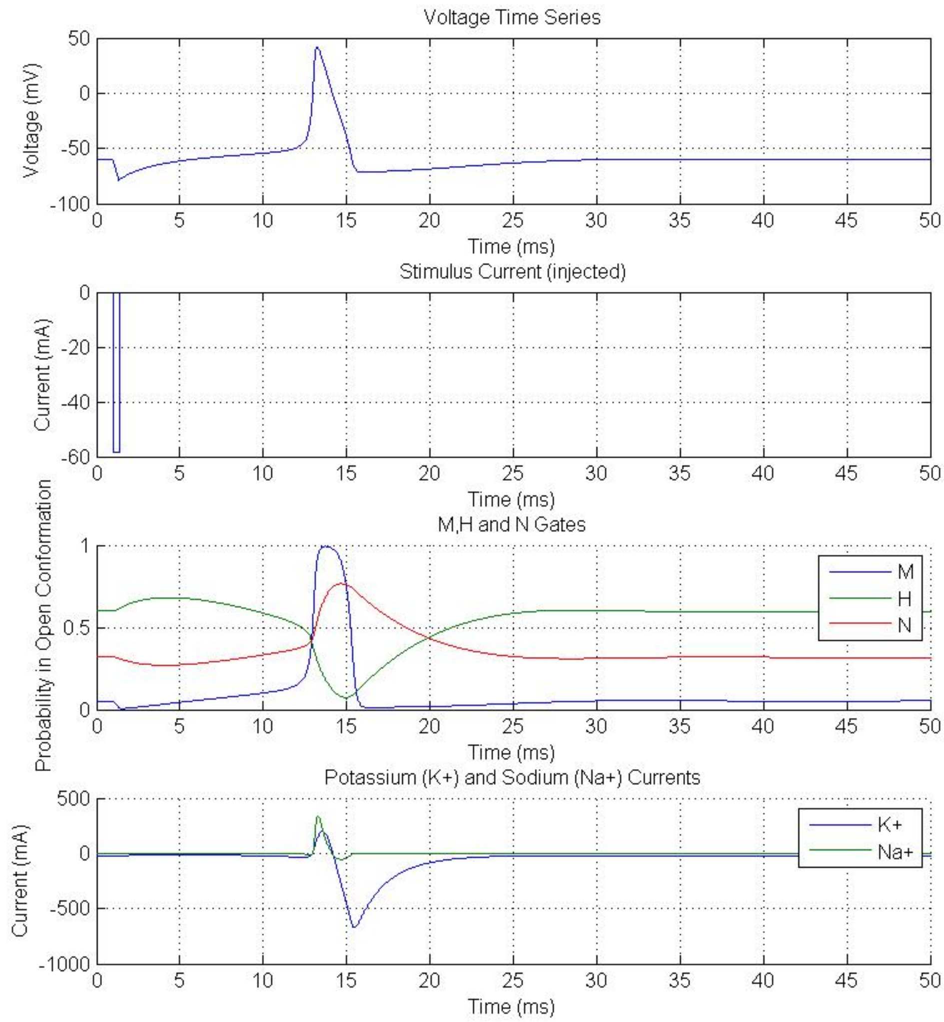

Figure 6: Minimum width of negated current pulse to elicit AP.