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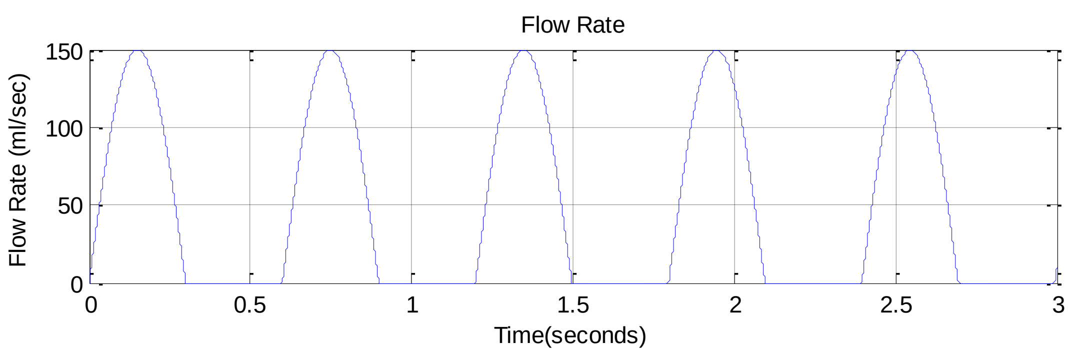
CardioVascular Mechanics

Modelling Cardiac Output using the two-element Windkessel Model

```
close all; clear all; clc;
Q_max= 150;
f=1.67;
t=linspace(0,3,10000);

Q_in = Q_max*(sin(2*pi*f*t)>0).*(sin(2*pi*f*t));
plot(t,Q_in);
title('Flow Rate');
xlabel('Time(seconds)'); ylabel('Flow Rate (ml/sec)');
```

Total Volume Flow into the windkessel system in one cycle = $0.31831 * Q_{max}$ ml.

Flow is generally not a symmetrical curve… it rises faster than it falls. But it is a reasonable approximation. (Slide 32 lecture 3)



Estimating lumped Resistance R

```
clear all; close all; clc;
mean_BSA=[0.625,1.375]; %m^2
HR = [100,73]; %bpm
AAD_BSA = [20.4,13.7]; %mmm/m^2

Sys_BP = [92,105];  %mmHG
Dias_BP = [47,61];  %mmHG
Avg_BP = 0.5*(Sys_BP+Dias_BP); %mmHg

VTI = [21.5, 24.3]; %cm
aortic_valve_area = pi*((mean_BSA.*AAD_BSA)/2).^2; %mm^2

Stroke_Volume = VTI.*(0.01.*aortic_valve_area); %ml or cm^3

Cardiac_output = (HR./60).*Stroke_Volume; %ml/sec

R = Avg_BP./Cardiac_output; %mmHg/(ml/sec)
```

R for the Windkessel model for the PS is 1.5191 mmHg/(ml/sec)

R for the Windkessel model for the MS is 1.0073 mmHg/(ml/sec)

Calculating Input Flow Rate and Pressure

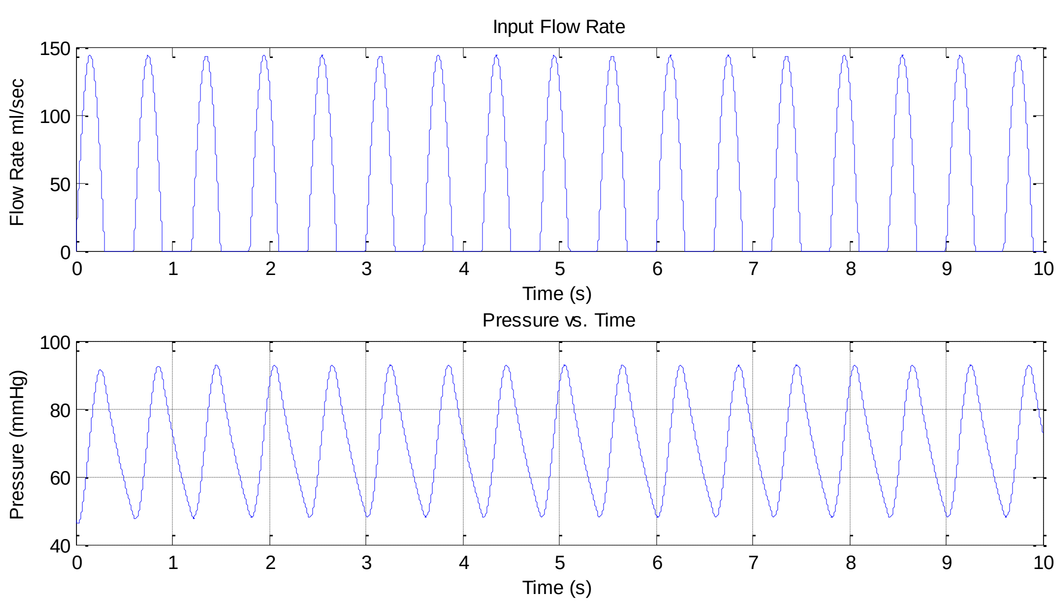
```
function [ dP ] = calculatePressure( t, P, R, C, f, Q_max )

Q_in = Q_max*(sin(2*pi*f*t)>0).*(sin(2*pi*f*t));

dP = (Q_in/C)-P/(R*C);

end
```

```
%calculate input flow rate and pressure for ten seconds with a periodic
%input flow of the form given in part a
clear all; close all; clc;
R = 1.5191; %mmHg/(ml/s)
f = 100/60; %heart rate expressed in Hz.
SV = 27.4504; %ml or cm^3
Q_max = SV*f/0.31831;
P_initial = 47; %mmHg
% C = 0.25; %compliance in ml/mmHg
C = 0.33;

t= linspace(0,10,10000);
Q_in = Q_max*(sin(2*pi*f*t)>0).*(sin(2*pi*f*t));

options = odeset('MaxStep',0.001);
tspan= [0 10];
[t_out, Pressure] = ode15s(@calculatePressure, tspan, P_initial,options, R, C, f, Q_max);

subplot(2,1,1);
plot(t, Q_in); title('Input Flow Rate')
ylabel('Flow Rate ml/sec'); xlabel('Time (s)');
subplot(2,1,2);
plot(t_out, Pressure); title('Pressure vs. Time');
xlabel('Time (s)'); ylabel('Pressure (mmHg)');
```

R = 1.5191 mmHg/(ml/sec)

C = 0.33 ml/mmHg

Q_{max}= 143.7 229 ml/sec



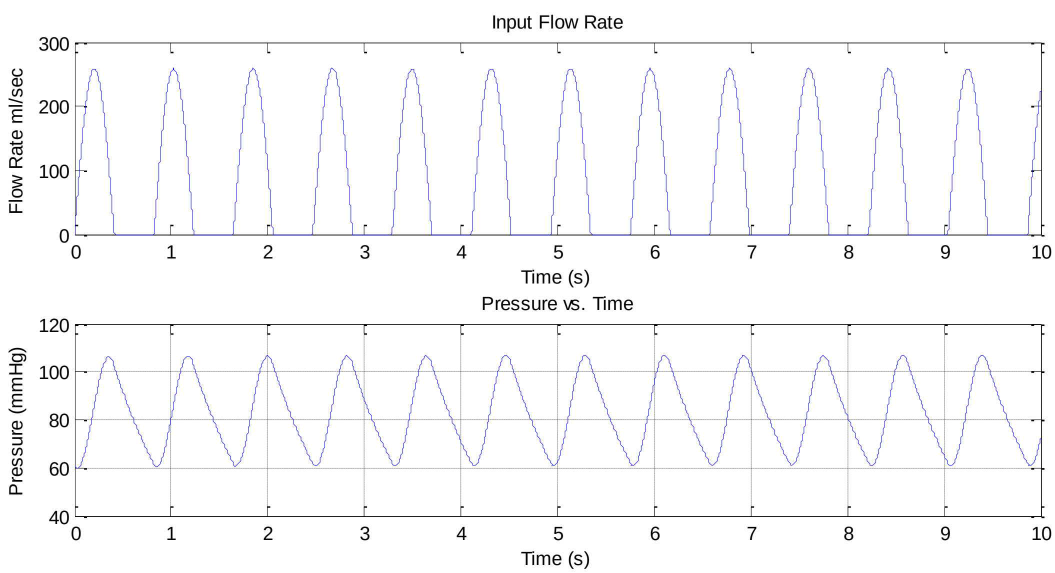
Compensating for Appropriate R and Recomputing Input Flow per Cycle

```
%calculate input flow rate and pressure for ten seconds with a periodic
%input flow of the form given in part a
clear all; close all; clc;
R = 1.0073; %mmHg/(ml/s)
f = 73/60; %heart rate expressed in Hz.
SV = 67.7240; %ml or cm^3
Q_max = SV*f/0.31831;
P_initial = 61; %mmHg
% C = 0.25; %compliance in ml/mmHg
C = 0.8;

t= linspace(0,10,10000);
Q_in = Q_max*(sin(2*pi*f*t)>0).*(sin(2*pi*f*t));

options = odeset('MaxStep',0.001);
tspan= [0 10];
[t_out, Pressure] = ode15s(@calculatePressure, tspan, P_initial,options, R, C, f, Q_max);

subplot(2,1,1);
plot(t, Q_in); title('Input Flow Rate')
ylabel('Flow Rate ml/sec'); xlabel('Time (s)');
subplot(2,1,2);
plot(t_out, Pressure); title('Pressure vs. Time');
xlabel('Time (s)'); ylabel('Pressure (mmHg)');
```

R = 1.0073 mmHg/(ml/sec)

C = 0.8 ml/mmHg

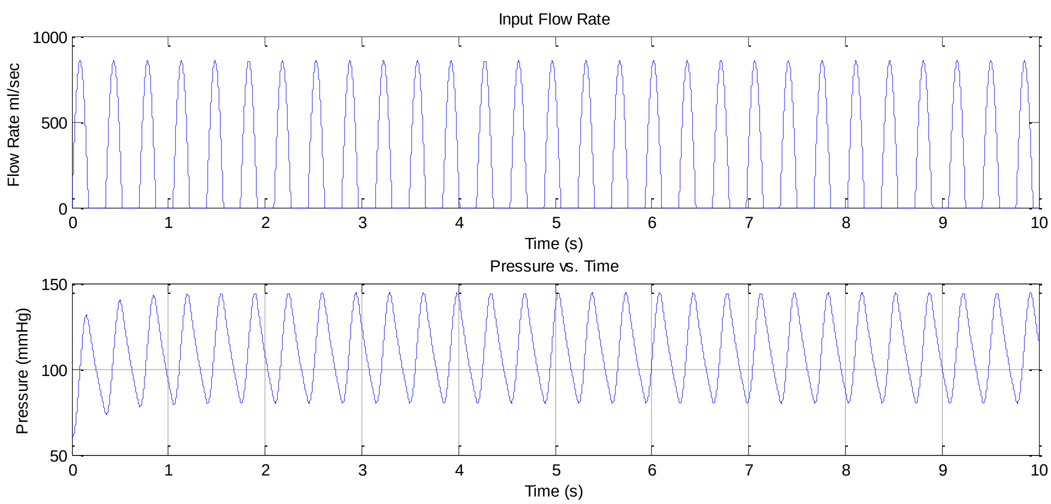
Q_{max}= 258.86 ml/sec



A larger time constant increases the period of the heartbeat. For the PS, the time constant was ~0.5013 and for the MS, the time constant was ~0.8058. That makes the ratio of the time constants 0.6221. The ratio of the periods is approximately 0.7125. The relationship between time constant RC and the period of the heartbeat seems to be roughly linear.

Effects of Exercise on Heart

```
close all;
clear all;
clc;

avg_BP = (149+73)/2; %mmHg
SV = 95; %ml
HR = 172; %bpm

Cardiac_output = (HR/60)*SV; %ml/sec
R = avg_BP/Cardiac_output; %mmHg/(ml/sec)
%resistance is less during peak exercise... this makes sense for increased
%blood flow to get oxygenated blood to the body.

%next part
f = 172/60; %heart rate expressed in Hz.
SV = 95; %ml or cm^3
Q_max = SV*f/0.31831;
P_initial = 61; %mmHg
% C = 0.25; %compliance in ml/mmHg
C = 0.8;

t= linspace(0,10,10000);
Q_in = Q_max*(sin(2*pi*f*t)>0).*(sin(2*pi*f*t));

options = odeset('MaxStep',0.001);
tspan= [0 10];
[t_out, Pressure] = ode15s(@calculatePressure, tspan, P_initial,options, R, C, f, Q_max);

subplot(2,1,1);
plot(t, Q_in); title('Input Flow Rate')
ylabel('Flow Rate ml/sec'); xlabel('Time (s)');
subplot(2,1,2);
plot(t_out, Pressure); title('Pressure vs. Time');
xlabel('Time (s)'); ylabel('Pressure (mmHg)');
```



R =0.4076 mmHg/(ml/s)

Exercising caused MS R to drop from 1.0073 to 0.476, a 59.54% decrease. This makes sense that resistance would be less during peak exercise, because of the need for increased blood flow to get oxygenated blood to the active body.

Using the compliance found before (0.8), the graph shows the max is 144 and minimum is 80. This is definitely in the ballpark, but not exactly 149/73 from the table. Decreasing C will increase the delta between max and min pressure. Therefore I'm assuming that C would need to increase slightly to get 140/73 .

Determining Values for different portions of the Mesenteric Bed

```
clear all; close all; clc;
N = [1 15 45];
radius = [0.15 0.05 0.03]; %cm
total_area = [0.07 0.12 0.13]; %cm^2
length = [6 4.5 3.9]; %cm
avg_vel = [16.8 10 9.3]; %cm/s
pressure_drop = [0.8 3.2 7.4];% mmHg

% Part a
total_Vol_Flow_Rate = total_area.*avg_vel; %cm^3/s

% Part b
blood_viscosity = 0.03; %Poise= gm/(cm*s)
Wall_Shear_Stress = 32.*blood_viscosity.*total_Vol_Flow_Rate./(8*pi*radius.^3); %dynes/cm^2

%part c
theoretical_resistance = (1/1333).*(128*blood_viscosity.*length./(16*pi.*radius.^4))./N; %mmHg/(cm^3/s)
computed_resistance = pressure_drop./total_Vol_Flow_Rate;
resistance_percent_error = 100.*(theoretical_resistance-computed_resistance)./theoretical_resistance;
```

Arteries	Total Vol Flow Rate (cm^3/s)	Wall Shear Stress ($dynes/cm^2$)	Theoretical Resistance ($mmHg/(cm^3/s)$)	Computed Resistance ($mmHg/(cm^3/s)$)	%difference between theoretical & computed
Mesenteric Artery	1.1760	13.3096	0.6792	0.6803	0.153 %
Large Branches	1.2000	366.6930	2.7509	2.6667	3.0615%
Small Branches	1.2090	1710.4	6.1319	6.1208	0.1824%

The volume flow rates are all within 2.73% of one another. This makes sense because in series formation, each arterial section should have the same volume flow rate.

For the wall Shear stress, the value 13.3096 for the mesenteric lines up pretty well with the average 15dynes/cm^2 as it is only an 11.27% decrease… the other ones are way larger (around 2344% increase and 1.1303e+04% increase respectively).

The resistance values were remarkably close with a maximum % difference between theoretical and computed being just over 3%.

Changes in resistance and blood pressure at different locations

```
%part d

clear all; close all; clc;
%constants
blood_viscosity = 0.03; %Poise= gm/(cm*s)

%table values
N = [1 15 45 1899 26640 238500 1051000 4.73*10^7];
radius = [0.15 0.05 0.03 6.8*10^-3 2.5*10^-3 1.55*10^-3 1.22*10^-3 4*10^-4]; %cm
radius_w_dilation =[0.15 0.05 0.03 1.05*6.8*10^-3 2.5*10^-3 1.55*10^-3 1.22*10^-3 4*10^-4]; %cm
total_area = [0.07 0.12 0.13 0.28 0.52 1.8 4.91 23.78]; %cm^2
length = [6 4.5 3.9 1.42 0.11 0.15 0.2 0.04]; %cm
avg_vel = [16.8 10 9.3 5.8 2.1 0.48 0.28 0.05]; %cm/s
pressure_drop = [0.8 3.2 7.4 23.5 7.2 5.4 8.1 2.4];% mmHg
%calculated values

%calcualte resistance in the intestinal branch before and after the
%dilation
int_branch_R_b4_dil = (1/1333).*(128*blood_viscosity.*length(4)/(16*pi*radius(4)^4))/N(4); %mmHg/(cm^3/s);
int_branch_R_aftr_dil =  (1/1333).*(128*blood_viscosity.*length(4)/(16*pi*(1.05*radius(4))^4))/N(4);
%mmHg/(cm^3/s);

%the percent decrease in resistance in the intestinal branches is as below
```

```
percent_decrease_in_R = (int_branch_R_b4_dil-int_branch_R_aftr_dil)*100/int_branch_R_b4_dil;


%2nd part
computed_resistance = (1/1333).*(128*blood_viscosity.*length./(16*pi.*radius.^4))./N; %mmHg/(cm^3/s)
computed_resistance_with_dilation = (1/1333).*(128*blood_viscosity.*length./(16*pi.*radius_w_dilation.^4))./N;
%mmHg/(cm^3/s)

total_vol_flow_rate_b4_dilation = sum(pressure_drop)/sum(computed_resistance);%in cm^3/s
total_vol_flow_rate_after_dilation = sum(pressure_drop)/sum(computed_resistance_with_dilation);%in cm^3/s

vol_flow_rate_increase = total_vol_flow_rate_after_dilation-total_vol_flow_rate_b4_dilation; %in cm^3/s

vol_flow_rate_percent_increase = 100*vol_flow_rate_increase / total_vol_flow_rate_b4_dilation; % in percent
```

Percent decrease in resistance of the intestinal branches in response to 5% dilation of intestinal branch radius is 17.7298%.

The total volume flow rate would increase 0.0936 cm^3/s from 1.1904 cm^3/s before the dilation to 1.2840 cm^3/s after the dilation. That is a 7.867% increase.

<u>If the Intestinal Branches Contract rather than Dilate</u>

```
N_2 = [1 15 45 26640 238500 1051000 4.73*10^7];
radius_2 = [0.15 0.05 0.03 2.5*10^-3 1.55*10^-3 1.22*10^-3 4*10^-4]; %cm
length_2 = [6 4.5 3.9 0.11 0.15 0.2 0.04]; %cm

new_VFR = 0.8*total_vol_flow_rate_b4_dilation;

dilated_radius_2the4th=(((1/1333)*(128*blood_viscosity*length(4)/(16*pi))/N(4)))/(sum(pressure_drop)/new_VFR -
sum((1/1333).*(128*blood_viscosity.*length_2./(16*pi.*radius_2.^4))./N_2));
dilated_radius = dilated_radius_2the4th^(1/4);

normal_radius = 6.8*10^-3;

percent_change =  100*(dilated_radius-normal_radius)/normal_radius;
```

The radius of the intestinal branches would contract from normal 0.0068 cm to 0.006 cm. This is a contraction of 11.1933% of the normal radius.